INTEGRATION OF DATA ASSIMILATION SUBSYSTEM INTO ENVIRONMENTAL MODEL OF HARMFUL SUBSTANCES PROPAGATION

Petr Pecha and **Radek Hofman**

Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic

Introduction

The goal of data assimilation (DA) is to provide analysis which relies on so called background field from a model forecast and observations. Other inputs to data assimilation process can be physical constraints on the problem or any additional prior knowledge not included in the model. Merging of these contending sources of information had shown to be very promising in many branches of contemporary Earth sciences. In data assimilation we try to adjust the model according to the measured values that represents the research effort to move from isolated model predictions towards reality. An automatic procedure for bringing observations into the model is called objective analysis. The major progress of objective analysis was achieved in the field of meteorological forecasting techniques that represent efficient tool in struggle with tendency to chaotic destruction of physical knowledge. Advanced assimilation methods are capable of taking into account measurement and model errors in the form of error covariance matrices.

Assimilation algorithms

Physical knowledge embodied within the model predictions usually enters into procedure of data assimilation as a vector of background field values $\mathbf{x}_f$ of dimension n (n = number of analyzed grid points × number of analyzed quantities).

Initial condition for the task of prediction of radiological situation evolution is in our case usually given as a result of Atmospheric Dispersion Model (ADM). Reliability of this initial value $\mathbf{x}_f$ (background field) can be improved by the assimilation process. Let vector of available measurements $\mathbf{y}_o$ have dimension p. Analysis in each grid point is represented by vector $\mathbf{x}_a$ of dim n. The common principle of objective analysis can be expressed by the relation

$$\mathbf{x}_a = \mathbf{x}_f + \mathbf{W}\mathbf{d} \qquad (1)$$

Eq. (1) expresses update step of data assimilation process. It says that we obtain analysis if we take background field vector and add to it product of matrix $\mathbf{W}$ and vector $\mathbf{d}$. $\mathbf{W}$ is weight matrix of dimension $n \times p$ and $\mathbf{d}$ is a vector of innovations given by differences between values of measurement and values of the model. Model values are known in given discrete points and if we want to know value of model in arbitrary point we need to use forward observation operator H, which transforms points from model space into measurement space, so the differences can be then evaluated as $\mathbf{d} = \mathbf{y} - H\mathbf{x}$.

Current list of assimilation algorithms implemented in assimilation subsystem of the HARP system follows.

Classical interpolation In case of interpolation methods, the analysis is constructed only upon measurements of an analyzed quantity, so we omit $\mathbf{x}_f$ in Eq. (1) which reduces to $\mathbf{x}_a = \mathbf{W}\mathbf{y}$ and $\mathbf{W}$ represents an interpolation operator.

Successive Corrections Method SCM can take into account prior information provided by a mathematical model but neither errors of model nor errors of measurements can be assumed there. Only an empirical expert knowledge can be introduced through a tuning parameter ϵ. The first iteration step of analysis $\mathbf{x}_a$ in j^{th} grid point can be obtained as

$$x_a^j = x_f^j + \frac{\sum_{k=1}^{K_j} w^{jk}(y_k - x_k^b)}{\sum_{k=1}^{K_j} w^{jk} + \epsilon^2} \qquad (2)$$

Coefficients w^{jk} are empirically determined weights. We sum products of the weights and differences between measurements y_k and model values in locations of measurements over all measurements in region of influence around the analyzed point ([4]).

Kalman Filter Kalman filter (KF) can provide us intermittent DA and also a prediction of further evolution of an analyzed quantity and its error covariance structure (ECS). Prediction of radiological situation on terrain has two steps and it follows assimilation procedure given by Eq. (1).

ECS is expressed in the form of error covariance matrices of model $\mathbf{P}_f^t$ and measurements $\mathbf{R}^t$. $\mathbf{H}$ is a linear operator. Correction of model according to measurements is called *data update* step of KF (Eq. (3), Eq. (4)) and gives us also information on ECS of analysis (Eq. (5)).

$$\mathbf{x}_a^t = \mathbf{x}_f^t + \mathbf{K}^t(\mathbf{y}_o^t - \mathbf{H}\mathbf{x}_f^t) \qquad (3)$$

$$\mathbf{K}^t = \mathbf{P}_f^t\mathbf{H}^T(\mathbf{R}^t + \mathbf{H}\mathbf{P}_f^t\mathbf{H}^T)^{-1} \qquad (4)$$

$$\mathbf{P}_a^t = (\mathbf{I} - \mathbf{K}^t\mathbf{H})\mathbf{P}_f^t \qquad (5)$$

The second step is called the *time update*. This step performs time evolution of an analyzed quantity via linear model $\mathbf{M}$ (Eq. (6)) and also evolution of its ECS (Eq. (7)). Output of the second step of KF is prediction $\mathbf{x}_f^{t+1}$ and information on error of this prediction $\mathbf{P}_f^{t+1}$.

$$\mathbf{x}_f^{t+1} = \mathbf{M}\mathbf{x}_a^t \qquad (6)$$

$$\mathbf{P}_f^{t+1} = \mathbf{M}\mathbf{P}_a^t\mathbf{M}^T + \mathbf{Q} \qquad (7)$$

These two steps can be iteratively repeated as long as new measurements are available ([1]).

Implementation

Architecture of HARP system Our aim is to develop modeling, simulation and educational tool with unified user-friendly graphical interface (GUI) for utilization in radiation protection (more in the [3]). For purpose of modeling of the propagation of radionuclides in atmosphere up to 100 kilometers from the source of pollution we use SGPM model which is capable of taking into account hourly meteorological forecast. In Fig. 1 we can see architecture of the system which is reviewed in detail in the extended abstract.

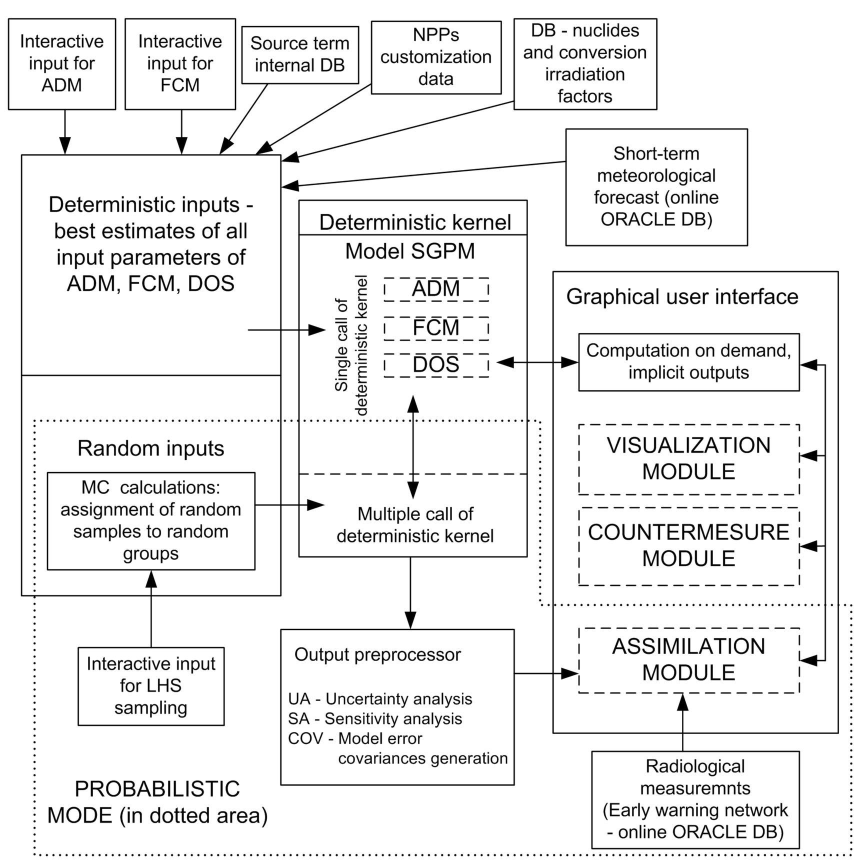

Figure 1. Block diagram of an architecture of the system.

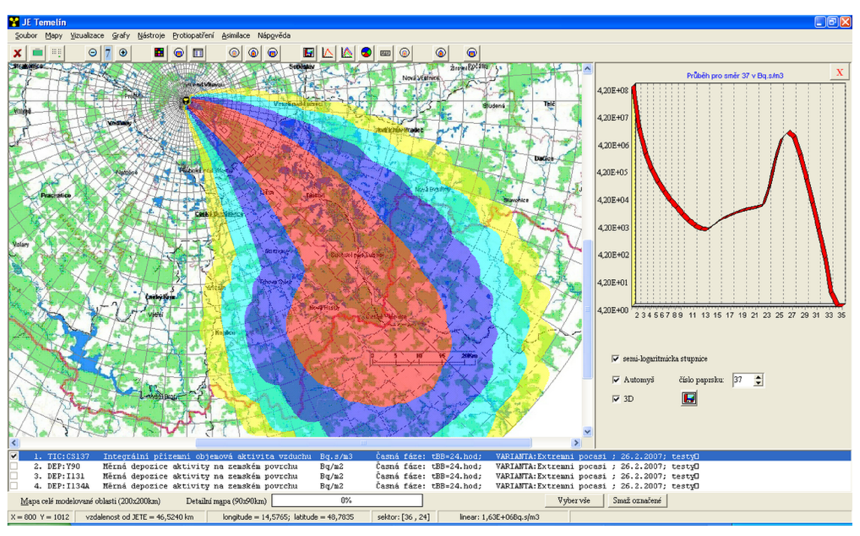

Figure 2. Main screen of the HARP system with scalable map and 1D graph showing values of an analyzed quantity along a certain radial direction.

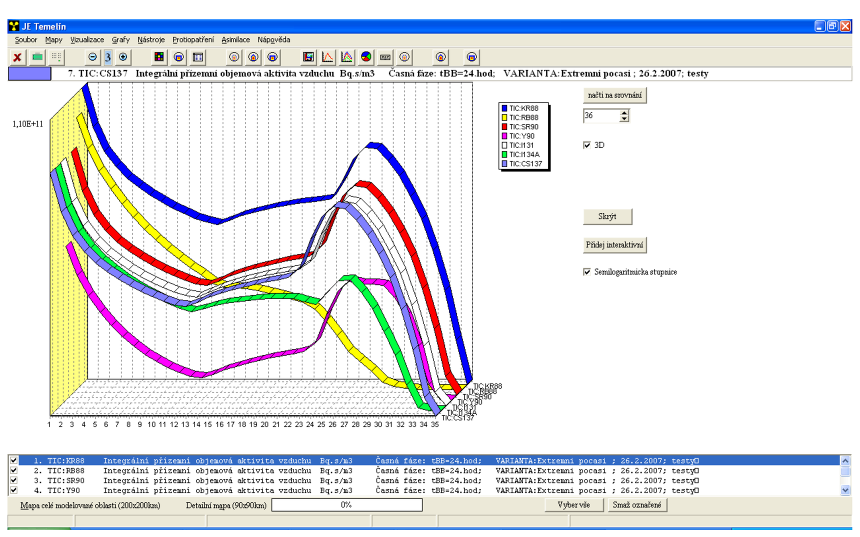

Figure 3. Comparative graphs of time-integrated concentration of different radionuclides.

Assimilation submodule Assimilation submodule offers GUI for interactive insertion of data and its maintenance and evaluation. Numerical and assimilation subsystems have direct bindings to visualization submodule, so both data and measurements can be visualized in GUI on relevant scalable map background (Fig. 2). Evaluation of results is also supported by data tables and comparative graphs (Fig. 3). Also access to ORACLE DB of meteorological forecasts and measurement stations included in Radiation Monitoring Network of the Czech Republic is established there. During testing of minimization methods a concept of "twin experiment" was used ([2]), where measured values are sampled from a random numerical model.

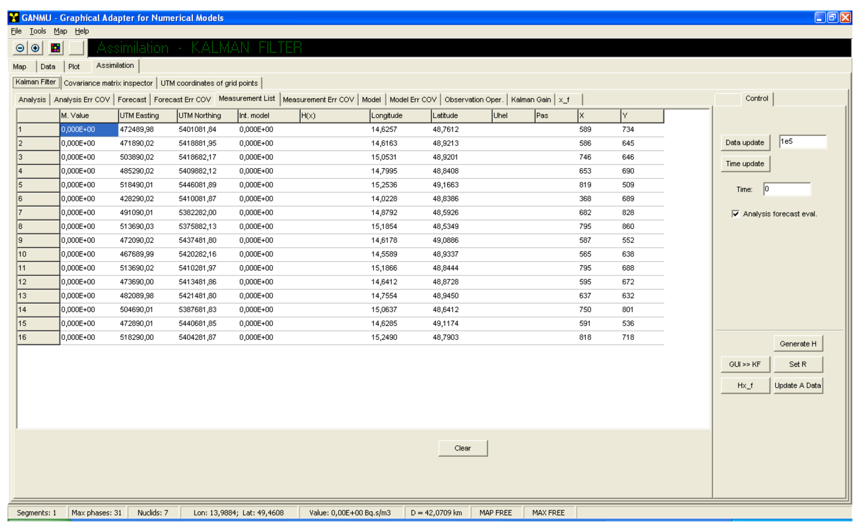

Figure 4. Screen from assimilation subsystem intended for managing of measurements assumed in assimilation process.

Results

Assimilation over local rain area

Influence of rain (acceptance of local character of a random input) In this example we assume Gaussian plume penetrating local "rain wall" between km 40 - 50 from the source (rain intensity $2mm \cdot hour^{-1}$). It results to intensive depletion of plume activity and increased deposition on terrain.

The only measurement was placed in the middle of the rain area. In Fig. 5 and 6 we can see the results for SCM (Eq. (2)) and the data update step of KF (Eq. (3)). SCM adjusted values in the whole region of influence ($10km$) towards measurement regardless of rain. In contrast, the KF correctly accounts for physical knowledge embodied in ECS values before, in and after the rain area.

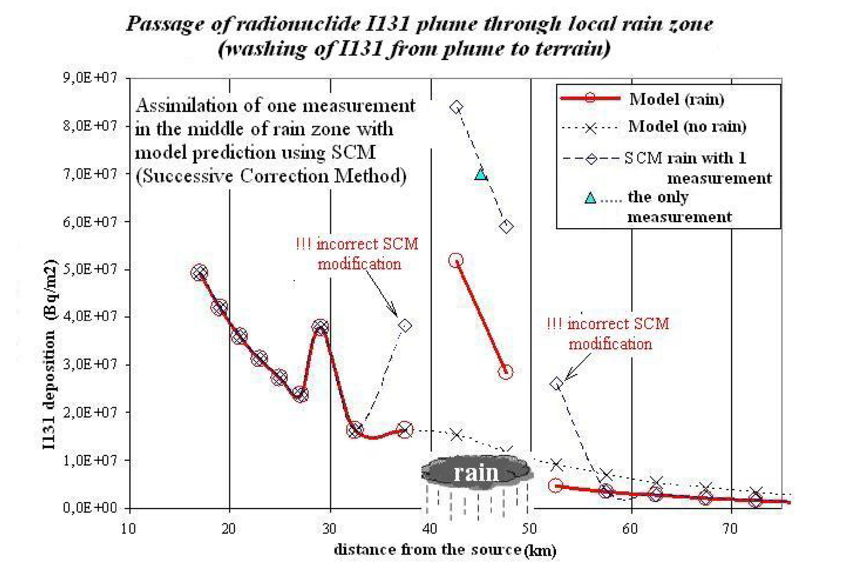

Figure 5. Assimilation result for SCM. SCM is "blind" with regard to local effect of random washing constant Λ.

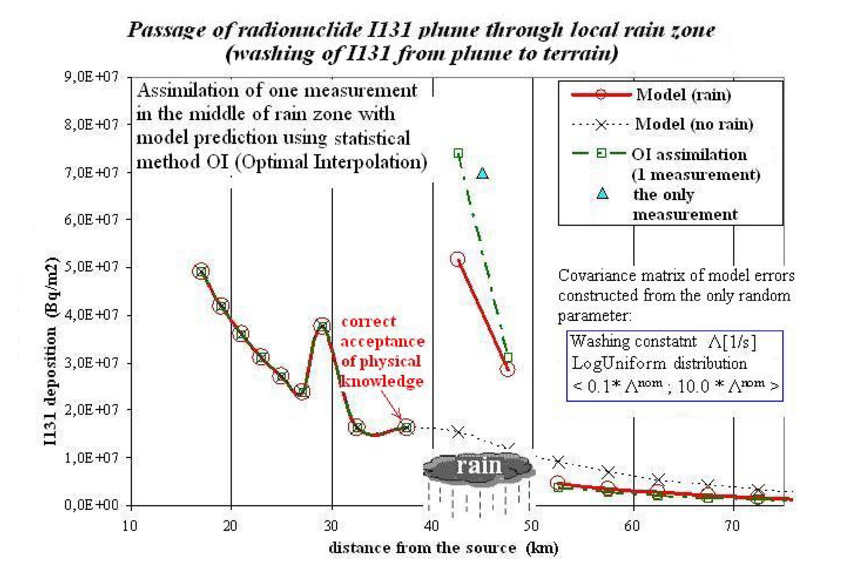

Figure 6. Assimilation result for KF.

Prediction of a long term deposition of ^{137}Cs

Utilization of KF for prediction of long-term ground deposition of ^{137}Cs Initial condition for assimilation is given by ADM when the plume is gone.

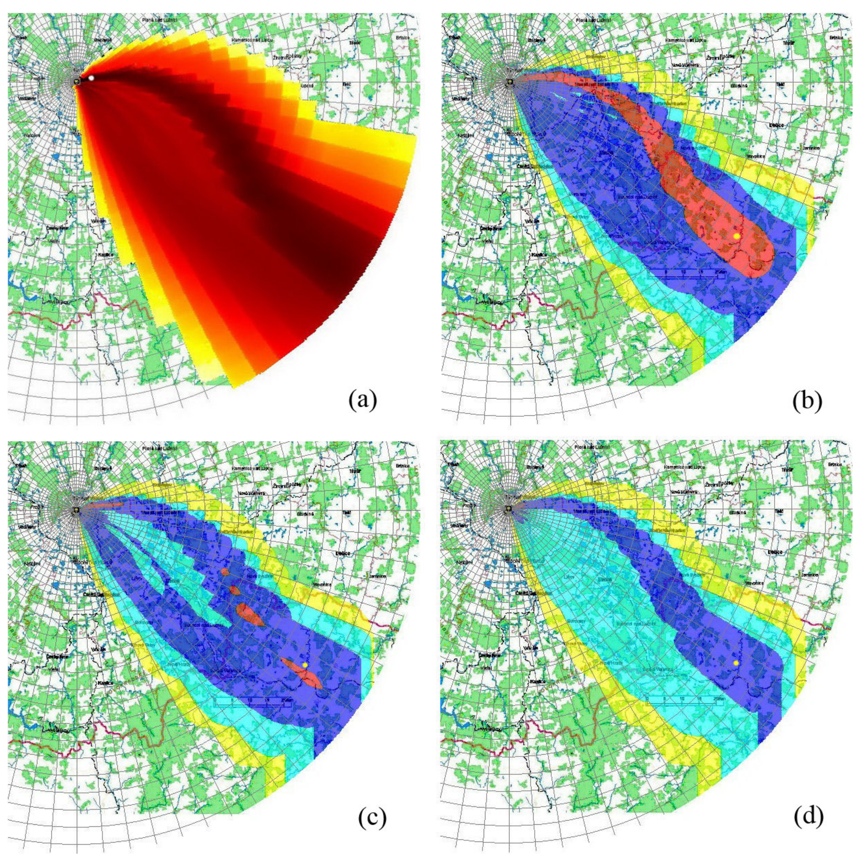

Figure 7. (a) Visualization of ECS for certain point (white point). (b) Background field from ADM. (c) BF after assimilation with the only measurement (yellow point). (d) Prediction of an evolution after 10 years.

As a model $\mathbf{M}$ in Eq. (6) was used Japanese model OSCAAR which assumes that decrease of radioactivity follows relation given as a superposition of two exponentials. The crucial task is estimation of ECS of the model and the background field. In this experiment we ECS estimated by Monte-Carlo approach as sample covariance of a drawn sample of size $N \approx 10^3$. Visualization of ECS for one certain analyzed point can be seen in the Fig. 7(a). In the Fig. 7(b) is background field and the Fig. 7(c) shows its correction by data update step of KF. There is the only measurement which was chosen to be five times smaller than the model value (the yellow point). Fig. 7(d) shows prediction of evolution of a radiation situation after 10 years (more in [5]).

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