Calculations of External Irradiation from Radioactive Plume in the Early Stage of a Nuclear Accident

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Abstract: A mathematical method for real-time calculations of cloudshine doses/dose rates used for purposes of online assimilation of model predictions with observations incoming from terrain is proposed. Model predictions of cloudshine doses have to be calculated simultaneously in an array of positions located on terrain around a nuclear facility. A modification of the classical straight-line Gaussian solution of the near-field dispersion problem is proposed. Movement of radioactive cloud driven by changes in meteorological conditions is described according to the segmented Gaussian scheme. Advanced $5/\mu$ concept based on integration up to five times mean free path of a γ photon ensures a quicker assessment of the doses of irradiation and predetermines the proposed procedure to be a proper tool for data assimilation analysis.

Keywords: Radioactivity release, photon fluence, cloudshine doses, data assimilation.

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Introduction

Decision-making staff should be provided with reliable predictions of time and space evolution related to contamination as early as possible. It can be achieved on the basis of optimal blending of informational resources including prior physical knowledge given by model, observations incoming from on-site locations, past experience, expert judgment and intuition. According to these prognoses required urgent emergency actions have to be planned and launched in the most impacted areas. Measured and calculated doses/dose rates of external irradiation from radioactive cloud are basic inputs to the objective

analysis of data assimilation techniques. Assimilation procedures require high-quality data from the measurement devices (sensors, alias receptors). The sensors are part of the Early Warning Network (EWN) and ground and airborne mobile monitoring groups. The EWN is represented by a teledosimetric system (TDS) located around the perimeter of a nuclear facility as well as an additional network of measuring devices at outer distances. Emergency preparedness procedures should account for integration of data from all types of measuring devices (fixed stations, apparatus deployed temporarily in case of emergency, monitoring vehicles, aerial monitoring etc.). The error structures of the model predictions and errors of incoming observations from terrain are processed by statistical assimilation techniques. Common principles of objective analysis for optimal blending of model predictions with observations include at first the update step using data incoming from terrain. An improved model prediction follows in the next time step using sophisticated advanced statistical methods based on Bayesian recursive tracking of the toxic plume progression. The corresponding likelihood function used within the Bayes' theorem relates the measured data to the model parameters. The significance of credible dose/dose rate estimation for the assimilation purposes is evident.

In this article a special algorithm is proposed for effectively estimating cloudshine doses/dose rates from radioactive cloud spread over idealised flat terrain. In the early phase of an accident, the pollution from the source passes through the ring of TDS sensors surrounding a nuclear facility. A recurrence scheme for substitution of the involved three-dimensional integration by a stepwise two-dimensional one is proposed. In the illustrative example we assume a ring of sensors on-site of a nuclear power plant (NPP) according to the sketch in Figure 3, below. The aim of this analysis is to simulate time evolution of sensor responses in all stages of the cloud propagation. The recursive corrections of the cloud drifting allow to predetermine the extent of dangerously affected areas more realistically.

Predictions of harmful admixtures spread at near distances from the source

We have adopted a classical solution of the diffusion equation for description of the initial phase of radioactive discharge drifting (near-field model). 3-D distribution of specific radioactivity concentration C^n of nuclide n in air [Bq.m^{-3}] is expressed by the straight-line Gaussian solution. The approach has a long tradition of use for dispersal predictions. Despite its simplicity, the Gaussian model is consistent with the random nature of turbulence and it is a solution of the Fickian diffusion equation for constant diffusivity coefficient K and average plume velocity $\bar{u}$. The model is tuned to the experimental data and offers a quicker estimation of the output values with minimum computation effort. Proved semi-empirical formulas are available for approximation of important effects like interaction of the plume with *nearby structures* and buoyant *plume rise* during release. Simplified formulas are used for power-law expression of wind speed *changes with height*, *depletion* of the plume activity due to *removal processes* of dry and wet deposition and decay, dependency on *physical-chemical forms* of admixtures and *land use characteristics*, simplified accounts of *inversion* meteorological situations and plume *penetration of inversion*, account for small changes in *surface elevation, terrain roughness* etc. (Hanna, Briggs, and Hosker, 1982).

It is evident, that a straight-line solution is limited to its use in short distances from the source up to several kilometres corresponding to the first hour (eventually half an

hour) of the short-term meteorological forecast. The plume drifting in subsequent meteorological phases (hours) is described by segmented Gaussian plume model (SGPM) (Hofman and Pecha, 2011). It takes into account the hourly (half-hourly) changes of meteorological conditions given by a short-term forecast provided by the meteorological service. Complicated scenarios of release dynamics are synchronized with the available meteorological forecast. The radioactivity concentration in the first hour of the plume propagation is given by a straight-line solution of the diffusion equation in the form:

$$C^n(x,y,z) = \frac{A^n}{2\pi \cdot \sigma_y(x) \cdot \sigma_z(x) \cdot \bar{u}} \cdot \exp\left(-\frac{y^2}{2\sigma_y^2(x)}\right) \left[\exp\left(-\frac{(z-h_{ef})^2}{2\sigma_z^2(x)}\right) + \exp\left(-\frac{(z+h_{ef})^2}{2\sigma_z^2(x)}\right) + \right.$$

(1)

$$\left. + \exp\left(-\frac{(z-2H_{mix}+h_{ef})^2}{2\sigma_z^2(x)}\right) + \eta_{JV}(z) \right] \cdot f_R^n(x) \cdot f_F^n(x) \cdot f_W^n(x)$$

$C^n(x,y,z)$ Specific activity of radionuclide n in spatial point (x,y,z) in $[Bq/m^3]$; x – direction of spreading; y, z – horizontal and vertical coordinates

$\sigma_y(x), \sigma_z(x)$ Horizontal and vertical dispersions at distance x from the source $[m]$; expressed by empirical formulas

A^n Release source strength of radionuclide n $[Bq/s]$; assumed constant within time interval

$\bar{u}$ Mean advection velocity of the plume in direction x $[m/s]$

h_{ef}, H_{mix} Effective height of the plume axis over the terrain $[m]$, height of the planetary mixing layer $[m]$

$\eta_{JV}(z)$ Effect of additional multiple reflections on the inversion layer/mixing height and on the ground (for this near-field model hereafter ignored)

f_R^n, f_F^n, f_W^n Plume depletion factors due to radioactive decay and dry and wet deposition - dependant on nuclide n and its physical-chemical form (aerosol, organic, elemental). The factors stand for "source depletion" approach introduced into the classical straight-line Gaussian solution. Release source strength at distance x is depleted according to $A^n(x, y=0,z=h_{ef}) = A^n(x=0, y=0,z=h_{ef}) \cdot f_R^n(x) \cdot f_F^n(x) \cdot f_W^n(x)$.

The exponential terms in equation (1) mean from left to right: the basic diffusion growth of the plume, its reflection in the ground plane, and its reflection from the top of the mixing layer H_{mix}.

## A new algorithm for fast evaluation of external irradiation from the cloud

We shall consider physical quantity of photon fluence which represents number of γ photons passing through a specific area. Transport of photons with energy E_γ to receptor R will be described by photon fluence rate $\Phi(E_\gamma, R)$ in units $(m^{-2}.s^{-1})$. External exposure from a finite plume can be estimated by applying traditional methods based on three-dimensional integration over the cloud (e.g. ADMS4, 2009) or on specially constructed three-dimensional columned space divided on many finite grid cells, e.g. (Wang, Ling,

and Shi, 2004). Photon fluence rate in the receptor point R from a point source with release strength A (Bq/s) is calculated according to equation (2a). Estimation of the fluence rate $\Phi(E_\gamma, R)$ from the whole plume based on the three-dimensional integration is given by the scheme (2b).

$$\Phi(E_\gamma, R) = \frac{A \cdot B(E_\gamma, \mu \cdot |\vec{r}|) \cdot \exp(-\mu \cdot |\vec{r}|)}{4\pi |\vec{r}|^2} \tag{2a}$$

$$\Phi_{total}(E_\gamma, R) = \iiint\limits_{V_{mrak}} \frac{f \cdot C(\vec{r}) \cdot B(E_\gamma, \mu \cdot |\vec{r}|) \cdot \exp(-\mu \cdot |\vec{r}|)}{4\pi |\vec{r}|^2} dV \tag{2b}$$

$B(E_\gamma, \mu \cdot |\vec{r}|)$ stands for build-up factor. We use either its linear form $B(E_\gamma, \mu \cdot |\vec{r}|) = 1 + k \cdot \mu \cdot r$ ($k = (\mu - \mu_a)/\mu_a$, μ and μ_a are linear and mass attenuation coefficients) or the alternative Berger formula. Comparison of the two options can be found in literature, e.g. in (Overcamp, 2007). f is the branching ratio to the specified energy E_γ . $|\vec{r}|$ is the distance between receptor point R and an element of the plume dV.

Activity concentration is given by equation (1) and its Gaussian shape is schematically illustrated in Figure 1. The continuous and constant release in direction of axis x with average velocity $\overline{u}$ is segmented into a number of elliptic discs according to Figure 1.
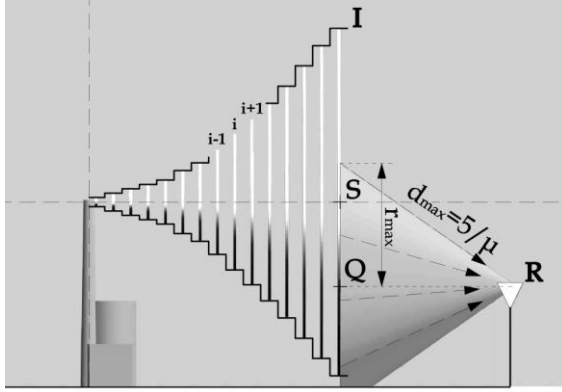
Thickness of the discs is selected as Δx = 10 m. The centre of the disc i reaches the position $x_i = (i - 1/2) \times \Delta x$ during $x_i / \overline{u}$ seconds. Lumped parameter technique is introduced when the model parameters are averaged within interval Δx on the disc i. Distribution of the activity concentration in the disc i on plane $x = x_i$ (the average value on Δx) is driven according to the straight-line solution given by equation (1)

Figure 1: Segmentation of the continuous release into a disc sequence. Point Q is a projection of the receptor point R to the plane of disc I; d_{max} , r_{max} relates to the integration boundaries based on "five times mean free path" concept (in more detail in Figure 2).

where the corresponding averaged disc parameters are substituted (e.g. x_i, $\sigma_y(x_i)$, $\sigma_z(x_i)$ and depletion $f_R(x_i) \cdot f_F(x_i) \cdot f_W(x_i)$ etc.). At this stage we have implemented the $5/\mu$ method which ensures substantial improvement of the calculation effectiveness. The $5/\mu$ concept (generally n/μ method – we have tested $n = 5,10,15$) imposes integration limit from the receptor R up to the distance $d_{max} = 5/\mu$. Integration boundary (see also bold-dashed integration circle in Figure 2) is formed by intersection of the cone (receptor R in the cone vertex) and the plane of the newest disc I. Only the points located inside contribute significantly to the fluence rate at R – more in (Wang, Ling and Shi, 2004). This accelerates computational speed and improves capability to run successfully the assimilation procedures in real time mode. It outperforms the computationally expensive

traditional methods based on full 3-D integration techniques (Raza, Avilla, and Cervantes, 2001). Substantial performance improvement predetermines the $5/\mu$ approach for its application during nuclear emergency situations (Wang, Ling, and Shi, 2004).

Replacement of traditional 3-D integration by a stepwise 2-D computational scheme based on the lumped parameter approach

Following the above considerations, we have assumed the external irradiation from the plume segment on interval $<x_i-\Delta x/2; x_i+\Delta x/2>$ to be substituted by an equivalent effect of the disc of thickness Δx with lumped model parameters on $<x_i-\Delta x/2; x_i+\Delta x/2>$. A lateral view of the segmented plume propagation is demonstrated in Figure 1. Let's analyse the contribution from the elliptical disc I from Figure 1 to the fluence rate at receptor R. The same situation is outlined in the front view in Figure 2.
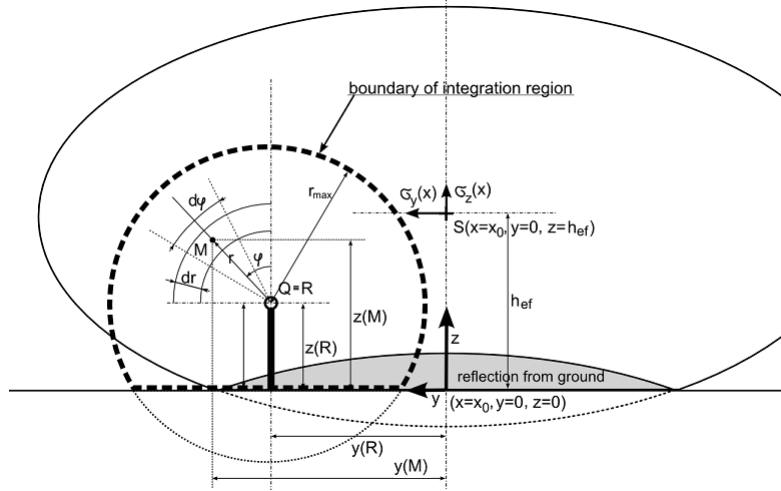

Figure 2: Frontal view from receptor point R to the elliptical disc I and circular integration region. The points Q, M, S lay on the plane of the disc I.

The boundary of integration region laying in the plane of disc I is based on $5/\mu$ approximation (bold dashed line composing part of circle above ground with radius r_{max} and centre in the point Q), $r_{max}^2 = (5/\mu)^2 - [x(R) - x(Q)]^2$. Contribution of the disc I to the photon fluence rate at receptor R is given by:

$$\Phi\big(E_\gamma, R, I\big) = \frac{\Delta x}{4\pi} \int_{r=0}^{r\,max} \int_{\varphi=0}^{2\pi} \frac{C^I\big(x_I; r, \varphi\big) B\big(E_\gamma, \mu.d\big) \exp\big(-\mu.d\big)}{d^2} \, r \, d\varphi \, dr \qquad (3)$$

Referring to Figure 2, d is the distance between points R(x(R), y(R), z(R)) and M(x(S), y(M), z(M)); $d^2 = (x(S) - x(R))^2 + (y(M) - y(R))^2 + (z(M) - z(R))^2$; $x(S) = x_I = (I-1/2)\times \Delta x$ is the distance of the centre of the disc I from the release point; $y(M) = r\times\sin(\varphi)$; $z(M) = z(R) + r\times\cos(\varphi)$. The equivalent mean activity concentration $C^I(x_I, y, z)$ in disc I is expressed using equation (1) where index n is omitted. Valid values of coordinate z should be positive, dispersion coefficients and depletion factors are calculated for position

x_I (i.e. time $x_I / \bar{u}$ seconds) in dependence on physical-chemical form of a nuclide. Equivalent disc source strength A (Bq.s^{-1}) substitutes the original discharge according to A (x_I, $y= 0,z = h_{ef}$) $=A$ ($x = 0$, $y = 0,z = h_{ef}$)$\cdot f_R (x_I) \cdot f_F (x_I) \cdot f_W (x_I)$. The values of photon fluence rates are successively stored into the array $F(1:N_{sens}, 1:I_{total})$. N_{sens} means the number of receptors being simultaneously taking into account, I_{total} stands for the total number of the 10-meter segments of the plume separation (so far selected value $I_{total} = 720$). Total fluence rates, total fluence and corresponding total cloudshine doses/dose rates are generated by summing up the values in all time steps. Taking $\Phi(E_\gamma, R, I)$ according to the equation (3), we examine two situations:

Continuous release of admixtures still lasts: The plume has reached position of the disc *I*. Propagation to the *I+1* disc is in progress. Contribution of each elemental disc $i = 1, ..., I$ to the fluence rate $\Phi(E_\gamma, R, i)$ at receptor R was calculated in the previous steps and stored in the array *F*. The new contribution $\Phi(E_\gamma, R, I+1)$ from disc *I+1* is calculated using integration (3). Recurrent formula for <u>overall fluence rate at receptor R</u> can be formally rewritten as:

$$\Phi\left(E_\gamma, R, i =1,...,I+1\right) = \Phi\left(E_\gamma, R, i =1,...,I\right) + \Phi\left(E_\gamma, R, I+1\right)$$

$$where \quad \Phi\left(E_\gamma, R, i = 1,...,I\right) = \sum_{i=1}^{i=I} \Phi\left(E_\gamma, R, i\right) \tag{4}$$

Then, only computation effort is needed for evaluation of 2-D integration of the latest disc *I+1*. Analogously, the recurrent formula for <u>entire photon fluence at receptor R</u> from the beginning of the release up to the disc *I+1* is given by:

$$\Psi\left(E_\gamma, R, i = 1,...,I+1\right) = \Psi\left(E_\gamma, R, i = 1,...,I\right) + \sum_{i=1}^{i=I+1} \Delta t_i \cdot \Phi\left(E_\gamma, R, i\right)$$

$$where \quad \Psi\left(E_\gamma, R, i = 1,...,I\right) = \sum_{i=1}^{i=I} \left[(I+1-i)\cdot \Delta t_i \cdot \Phi\left(E_\gamma, R, i\right)\right] \tag{5}$$

$\Delta t_i = \Delta t = \Delta x / \bar{u}$ seconds

Release terminated, propagation continues: The plume has reached the position of the disc I just at the moment when the release has terminated. Propagation continues to the discs positions *I+1, I+2, ..., I+j*. Fluence rate $\Phi(E_\gamma, R, I+j+1)$ for position *I+j +1* is calculated from the previous position *I+j* according to the following recurrent formula:

$$\Phi\left(E_\gamma, R, i = 1,...,I+j+1\right) =$$

$$= \Phi\left(E_\gamma, R, i = 1,...,I+j\right) + \Phi\left(E_\gamma, R, I+j+1\right) - \Phi\left(E_\gamma, R, j+1\right) \tag{6}$$

Hence, contribution from the leftmost disc of a parcel is skipped; the new rightmost one is calculated. Similarly, the recurrent expression for the total fluence Ψ has form:

$$\Psi\left(E_\gamma, R, i = 1,...,I+j+1\right) = \Psi\left(E_\gamma, R, i = 1,...,I+j\right) + \sum_{i=j+1}^{i=I+j+1} \Delta t_i \cdot \Phi\left(E_\gamma, R, i\right) \tag{7}$$

The photon fluence rate given by expression (3) is integrated numerically using Gauss-Legendre integration formula which is the most commonly used form of Gaussian quadratures. Verification of the numerical algorithm was done for simplified geometry of emitting disc without absorption and build-up effect (for this case, the equation (3) can be integrated analytically - simplified experiment for "irradiation in vacuum").

Simulation of responses from a sensor network

Here we present the responses on 40 sensors surrounding the NPP according to Figure 3 consisting of a ring of 24 TDS sensors on perimeter of a NPP with distances roughly about 450 meters from a hypothetical source. The rest of sensors are situated at larger distances inside the emergency planning zone (roughly up to 15 km). Our approach is demonstrated on a release of nuclide ^{131}I with source strength $2.5 \cdot 10^{+11}$ Bq.s^{-1}. Time evolution of fluences/ fluence rates and cloudshine doses/dose rates
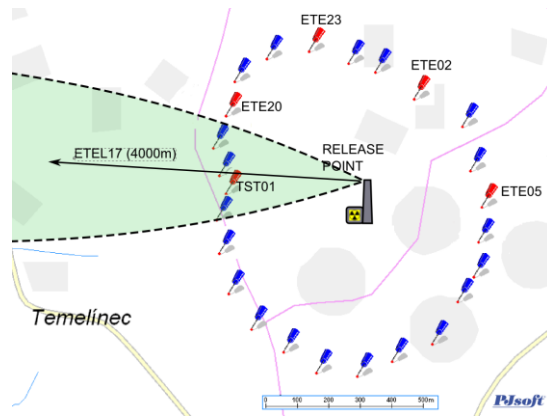

Figure 3: Sketch of configuration of a sensor network around a nuclear facility.

from continuous release of ^{131}I are simulated at all 40 sensors at one time. Effective height of release h_{ef} is 45 m. Pasquill category of atmospheric stability F and wind velocity in 10 m height u_{10}=1.0 m.s^{-1} are assumed. The results for sensors TST01 resp ETEL17 (roughly 400 meters resp 4 000 m in direction of the plume propagation – see Figure 3) are illustrated in Figure 4. The method n/μ was examined for $n = 5$ and $n = 10$, but the differences are too small (no more than 1%) to be visualised in Figure 4. The calculations for $n = 5$ are more than two times faster in comparison with $n = 10$.
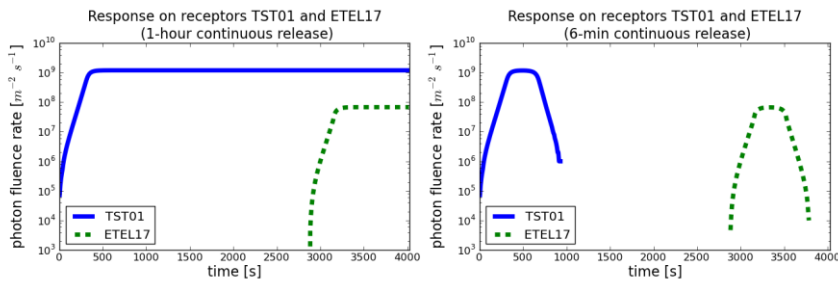

Figure 4: Photon fluence rate on receptors during the plume drifting. Left: Continuous one-hour release (up to 4025 s ~ 4960 meters). Right: Spreading of smaller plume of 6 min duration.

The contributions of reflections from the ground and top of the mixing layer were also estimated. The values of fluence rates at ground-level sensors are more than two times lower when the reflections are incorrectly neglected. Time evolution of responses in dose rates and doses on 5 receptors (sensors) TST01, ETE23, ETE05, ETE20 and ETE02 (selected from TDS ring in Figure 3) is illustrated in Figure 5. It relates to the short continuous release of ^{131}I with source strength $2.5 \cdot 10^{+11}$ Bq.s^{-1} with duration of 6 minutes.

The irradiation dose rate at a certain receptor R is labelled as $H(E_\gamma, R, I)$ and is expressed in units gray per second (Gy.s^{-1}). We are using the expression for monoenergetic photons with energy E_γ emitted from the disc I in the form:

$$H(E_\gamma, R, I) = \frac{\omega \cdot K \cdot \mu_a \cdot E_\gamma}{\rho} \cdot \Phi(E_\gamma, R, I) \tag{8}$$

The fluence rate $\Phi(E_\gamma, R, I)$ is given by equation (3). Conversion factor $K = 1.6 \cdot 10^{-13}$ Gy.kg.Mev^{-1} ; $\omega = 1.11$ is a ratio of absorbed dose in tissue to the absorbed dose in air, air density $\rho = 1.293$ kg.m^{-3}. For nuclide ^{131}I $E_\gamma = 0.3625$ MeV (γ yield is taken to be 100 %), linear attenuation coefficient $\mu = 1.40531 \cdot 10^{-2}$ (m^{-1}), mass attenuation coefficient $\mu_a = 3.30969 \cdot 10^{-3}$ (m^2.kg^{-1}). The absorbed dose in human tissues are calculated according to equation (8), when substituting the total fluence Ψ from equations (5) or (7) instead of the fluence rate Φ .
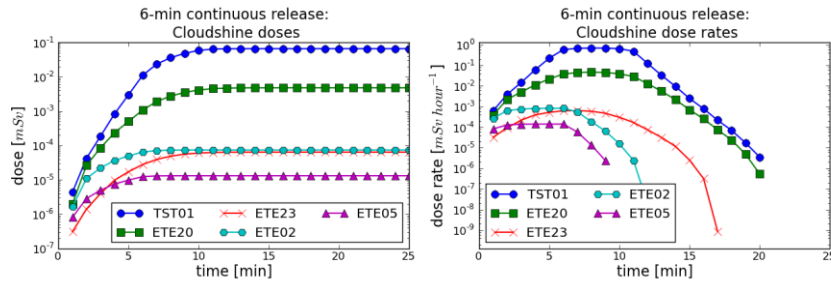

Figure 5. Responses on receptors (doses and dose rates) from smaller 6-minute release duration. Five receptors are selected according to Figure 3.

Common practice in the radiation protection field is to multiply absorbed doses by relative biological effectiveness factor F_q which accounts for different biological damage with regards to different types of ionizing radiation. The corresponding radiological quantity is expressed in Sv (sieverts). The factor F_q for photons equals 1.0.

Conclusion

Fast algorithm is presented for an evaluation of external irradiation from the radioactive cloud drifting over idealised flat terrain. Network of sensors surrounding a nuclear facility can include both fixed stations and various receptors located on possible aerial and surface monitoring vehicles. The algorithm is designed for support of computationally expensive assimilation methods based on particle filtering techniques. Another significant field of application can be an optimisation of configuration of environmental monitoring networks for emergency management and verification of their

detection abilities. In both cases, an estimation of irradiation doses in many tens of points (receptors) around the source of pollution can be accomplished quickly (at a stroke), so that the successive assimilation of model predictions with the measurements could be managed in real time. An implementation of more realistic scenario conditions (large nuclide mixtures, estimation of deposition fractions, modelling behind the emergency planning zone at distances of more than 15 kilometres) are in progress. A weak point of the applied Gaussian scheme is limited capability to account for the effect of shielding by the near-standing objects. Only rough semi-empirical compensations based on practical experience and wind-tunnel observations are available, e.g. (Hanna, Briggs, and Hosker, 1982). These shielded areas should be treated specifically and with care. For these purposes, transition to the more sophisticated techniques of the dispersion modelling is essential, e.g. Lagrangian particle models (Armand et al., 2005). The originated plume segmentation method based on the lumped parameter approach according to Figure 1 with methodical extension to the medium range distances can be proposed as a basic scheme for formulation of a certain dispersion scheme alternative to the standard puff model.

Acknowledgments

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